Indian Statistical Institute Admission Test for B.Math/B.Stat 2016

May 8, 2016

Q1

In a sports tournament of n players, each pair of players plays exactly one match against each other. There are no draws. Prove that the players can be arranged in an order $P_1, P_2, ..., P_n$, such that P_i defeats P_{i+1} $\forall$ $i = 1, 2, ..., n-1$.

Q2

Consider the polynomial $ax^3 + bx^2 + cx + d$, where ad is odd and bc is even. Prove that all roots of the polynomial cannot be rational.

Q3

$P(x) = x^n + a_1x^{n-1} + ... + a_n$ is a polynomial with real coefficients. $a_1^2 < a_2$. Prove that all roots of $P(x)$ cannot be real.

Q4

Let $ABCD$ be a square. Let A lie on the positive x-axis and B on the positive y-axis. Suppose the vertex C lies in the first quadrant and has co-ordinates (u, v). Then find the area of the square in terms of u and v.

Q5

Prove that there exists a right-angled triangle with rational sides and area d iff there exist rational numbers x, y, z such that x^2, y^2, z^2 are in arithmetic progression with common difference d.

Q6

Suppose in a $\triangle ABC$, A, B, C denote the three angles and a, b, c denote the three sides opposite to the corresponding angles. Prove that, if $sin\ (A - B) = \frac{a}{a+b} \sin A \cos B - \frac{b}{a+b} \sin B \cos A$, then $\triangle ABC$ is isosceles.

Q7

f is a differentiable function, such that $f(f(x)) = x$, where $x \in [0, 1]$. Also, $f(0) = 1$. Find the value of

$$\int_0^1 (x - f(x))^{2016} dx$$

Q8

$(a_n)_{n \geq 1}$ is a sequence of real numbers satisfying $a_{n+1} = \frac{3a_n}{2+a_n}$.
(i) If $0 < a_1 < 1$, then prove that the sequence a_n is increasing and hence,

$$\lim_{n \to \infty} a_n = 1$$

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(ii) If $a_1 > 1$, then prove that the sequence a_n is decreasing and hence,

$$\lim_{n \to \infty} a_n = 1$$

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